WS 13 — Unit 2 Review

1) $f(x)=\begin{cases} x^2, & x<1 \\ 3x-2, & x\geq 1 \end{cases}$

CONT

$f(1)=1$ ✓

$\lim_{x\to 1^-} x^2=1$ $\lim_{x\to 1^+}(3x-2)=1$ ✓

Diff

Left

$\lim_{x\to 1^-}\frac{x^2-1}{x-1}=\lim_{x\to 1^-}(x+1)=2$

Right

$\lim_{x\to 1^+}\frac{(3x-2)-1}{x-1}=\lim_{x\to 1^+}\frac{3x-3}{x-1}=3$

$f(x)$ is not diff. @ $x=1$

2) $y=x^4(4\sqrt{x}-3\sqrt[3]{x})$

$y'=4x^3(4\sqrt{x}-3\sqrt[3]{x})+x^4\left(\frac{2}{\sqrt{x}}-\frac{1}{\sqrt[3]{x^2}}\right)$

3) $f(x)=\frac{x^2+3x}{x-2}$

$f'(x)=\frac{(x-2)(2x+3)-(x^2-3x)}{(x-2)^2}$

4) $y=(x^2-2x+1)^3$

$y'=3(x^2-2x+1)^2\cdot(2x-2)$

$y''=6(x^2-2x+1)(2x-2)+3(x^2-2x+1)^2(2)$

5) $f(x)=x^3-5x+2$; $x=-2$

point: $(-2,4)$ slope: $f'(x)=3x^2-5$, $f'(-2)=7$

T: $y-4=7(x+2)$

N: $y-4=-\frac{1}{7}(x+2)$

6) $g(x)=x^2+3x+4$ $4x-y=7$

$y=4x-7$

$g'(x)=2x+3$

$2x+3=4$

$x=\frac{1}{2}$

$g(\frac{1}{2})=\frac{1}{4}+\frac{3}{2}+4=\frac{23}{4}$

$\boxed{(\frac{1}{2},\frac{23}{4})}$

7) $f(x)=\sqrt{x^2-2x}$ $x=3$

point: $(3,\sqrt{3})$ slope: $f'(x)=\frac{1}{2}(x^2-2x)^{-1/2}\cdot(2x-2)$

$f'(3)=\frac{2}{\sqrt{3}}$

T: $y-\sqrt{3}=\frac{2}{\sqrt{3}}(x-3)$

N: $y-\sqrt{3}=-\frac{\sqrt{3}}{2}(x-3)$

8) $A=f+2g$

$A'=f'+2g'$

$A'(3)=f'(3)+2g'(3)=\boxed{3}$

9) $B=f\cdot g$

$B'=f'g+g'f$

$B'(2)=f'(2)g(2)+g'(2)f(2)$

$=(3)(1)+(-2)(5)$

$=\boxed{-7}$

10) $C = \frac{g}{f}$

$C' = \frac{fg' - gf'}{f^2}$

$C'(1) = \frac{(3)(-3) - (3)(2)}{9}$

$= \boxed{-\frac{15}{9}}$

11) $D = \frac{1}{g} = g^{-1}$

$D' = -\frac{1}{g^2} \cdot g'$

$D'(0) = -\frac{1}{[g(0)]^2} \cdot g'(0) = \frac{4}{25}$

12) $E = \frac{f}{g}$

$E' = \frac{gf' - fg'}{g^2}$

$E'(3) = \frac{g(3)f'(3) - f(3)g'(3)}{[g(3)]^2}$

$E'(3) = \frac{0 + 10}{0} \rightarrow \not\exists$

13) $y = x^4(4\cos x - 3\tan x)$

$y' = x^4(-4\sin x - 3\sec^2 x) + 4x^3(4\cos x - 3\tan x)$

14) $f(x) = \frac{\cos x}{1 + \tan x}$

$f'(x) = \frac{(1+\tan x)(-\sin x) - \cos x(\sec^2 x)}{(1+\tan x)^2}$

15) $y = x^7(3\csc x)$

$y' = 21x^6 \csc x - 3x^7 \csc x \cot x$

$y'' = 126x^5 \csc x - 21x^6 \csc x \cot x - \left[3x^7(-\csc^3 x - \csc x \cot^2 x) + 21x^6 \csc x \cot x\right]$

16) $y = \frac{\tan x}{2x + \csc x}$

$y' = \frac{(2x + \csc x)(-\sec^2 x) - \tan x(2 - \csc x \cot x)}{(2x + \csc x)^2}$

17) $y = \sin(3x - 4)$

$y' = 3\cos(3x - 4)$

18) $f(x) = \tan^3(4x^5 - 2x)$

$f'(x) = 3\left[\tan(4x^5 - 2x)\right]^2 \cdot \sec^2(4x^5 - 2x)(24x^5 - 2)$

19) $y = (x^3 + 2)^4(\cot x - 2x)^5$

$y' = (x^3+2)^4 \cdot 5(\cot x - 2x)^4(-\csc^2 x - 2) + (\cot x - 2x)^5 \cdot 4(x^3+2)^3 \cdot (3x^2)$